Lecture 5: Electric Potential

Today:

- More applications of Gauss Law

- Start Ch 23: Reminder: Work and potential energy. How to calculate the electric potential energy of a collection of charges.

- The meaning and significance of electric potential.

- How to calculate it for: 1. collection of point charges, 2. charged sphere, 3. two oppositely charged planes.

- How to use equipotential surfaces to visualize how the electric potential varies in space.

- How to use electric potential to calculate the electric field.

- How to calculate electric potential in general
Field of an infinite plane sheet of charge

1. Choose a Gaussian surface with the same symmetry (the tricky step)

2. On any point on (one of the several pieces of) the surface the flux is constant => It is proportional to the magnitude of the field and to the area of this piece.

3. Divide the r.h.s. by the area => we get the field on any point of the surface.

• Example: A thin, flat, infinite sheet with a uniform surface charge density $\sigma$. Because of the symmetry $\vec{E}$ is perpendicular to the surface. No flux through the side walls! The flux through each cap is the same: $EA$. Total flux:
  
  $2EA = \sigma A/\varepsilon_0$

  Symmetric charge distribution => E field with the same symmetry

  $E = \frac{\sigma}{2\varepsilon_0}$
Consider a solid conductor. Assume that all the electric charges on the conductor or outside of it (in the environment) are static: they are not moving. Then \( \mathbf{E} \) anywhere also doesn’t change with time.

This is called an **electrostatic configuration**.

Conductors have enormous amount of charges (usually electrons \( e^- \)) that are free to move at the smallest electrostatic force acting on them.

If they are not moving: \( \mathbf{E} = 0 \) **inside** the conductor.

Take a small Gaussian sphere entirely inside.

Because \( \mathbf{E} = 0 \), the flux trough it is 0. The sphere is so small that it is pointlike.

**Gauss Law** tells us that the net charge at any point **inside** the conductor is 0.

Of course, in the conductor there are enormous number of \( e^- \) and positive ions which have charge exactly equal (magnitude) to the negative charge of the \( e^- \).

\( \mathbf{E} \) may be non-zero outside of the conductor. There can be net charge on the **surface** of the conductor.
Charges on conductors

• Suppose we place a small body with a charge $q$ inside a cavity within a conductor. The conductor is uncharged and is insulated from the charge $q$.

• According to Gauss’s law the total there must be a charge $-q$ distributed on the surface of the cavity, drawn there by the charge $q$ inside the cavity.

• The total charge on the conductor must remain zero, so a charge $+q$ must appear on its outer surface.

For $\vec{E}$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$. 
Field at the surface of a conductor

- Gauss’s law can be used to show that the direction of the electric field at the surface of any conductor is always perpendicular to the surface.
- The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density $\sigma$.

**Electric field at surface of a conductor, $\vec{E}$ perpendicular to surface**

$$E_\perp = \frac{\sigma}{\varepsilon_0}$$

Surface charge density

Electric constant

Next: some interesting applications that we didn’t have time to discuss.
We now consider Faraday’s historic icepail experiment.

We mount a conducting container on an insulating stand.

The container is initially uncharged.

Then we hang a charged metal ball from an insulating thread, and lower it into the container.
Faraday’s icepail experiment: Slide 2 of 3

• We lower the ball into the container, and put the lid on.

• Charges are induced on the walls of the container, as shown.

Charged ball induces charges on the interior and exterior of the container.
• We now let the ball touch the inner wall.

• The surface of the ball becomes part of the cavity surface, thus, according to Gauss’s law, the ball must lose all its charge.

• Finally, we pull the ball out; we find that it has indeed lost all its charge.

Once the ball touches the container, it is part of the interior surface; all the charge moves to the container’s exterior.
The Van de Graaff generator

- The **Van de Graaff generator** operates on the same principle as in Faraday’s icepail experiment.

- The electron sink at the bottom draws electrons from the belt, giving it a positive charge.

- At the top the belt attracts electrons away from the conducting shell, giving the shell a positive charge.

- Some belts move up positive charges, others (like the one in class) – negative. (The paddle rotates if it shoots out electrons e\(^{-}\).)
Electrostatic shielding

- A conducting box is immersed in a uniform electric field.
- The field of the induced charges on the box combines with the uniform field to give zero total field inside the box.
Electrostatic shielding

• Suppose we have an object that we want to protect from electric fields.

• We surround the object with a conducting box, called a Faraday cage.

• Little to no electric field can penetrate inside the box.

• The person in the photograph is protected from the powerful electric discharge.
Electric potential energy in a uniform field

• In the figure, a pair of charged parallel metal plates sets up a uniform, downward electric field.

• The field exerts a downward force on a positive test charge.

• As the charge moves downward from point $a$ to point $b$, the work done by the field is independent of the path the particle takes.

$$ W_{a\rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F \, dy = -(U_b - U_a) = -\Delta U $$

$$ W_{a\rightarrow b} = \Delta K, \Delta K = -\Delta U, \Delta K + \Delta U = 0 $$

$$ \Delta E = 0, \quad E = K + U $$

The work done by the electric force is the same for any path from $a$ to $b$: $W_{a\rightarrow b} = -\Delta U = q_0 Ed$
A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the field does *positive* work on the charge.

- The potential energy *decreases*.

Positive charge $q_0$ moves in the direction of $\vec{E}$:
- Field does *positive* work on charge.
- $U$ decreases.
A positive charge moving in a uniform field

- If the positive charge moves opposite the direction of the field, the field does *negative* work on the charge.
- The potential energy *increases*.

Positive charge $q_0$ moves opposite $\vec{E}$:
- Field does *negative* work on charge.
- $U$ increases.

$\vec{F} = q_0 \vec{E}$
A negative charge moving in a uniform field

- If the negative charge moves in the direction of the field, the field does *negative* work on the charge.

- The potential energy *increases*.

\[ \vec{F} = q_0 \vec{E} \]

Negative charge \( q_0 \) moves in the direction of \( \vec{E} \):
- Field does *negative* work on charge.
- \( U \) increases.
A negative charge moving in a uniform field

- If the negative charge moves opposite the direction of the field, the field does positive work on the charge.

- The potential energy decreases.

  Negative charge \( q_0 \) moves opposite \( \vec{E} \):
  - Field does positive work on charge.
  - \( U \) decreases.
Electric potential energy of two point charges

- The work done by the electric field of one point charge on another does not depend on the path taken.

- Therefore, the electric potential energy only depends on the distance between the charges.

\[
W_{a \rightarrow b} = \int_{a}^{b} \vec{F} \cdot d\vec{l} = \int_{a}^{b} F \cos \phi \, dl = \int_{a}^{b} F \, dr = -(U_b - U_a)
\]
Electric potential energy of two point charges

- The electric potential energy of two point charges only depends on the distance between the charges.

\[ U = \frac{1}{4\pi \varepsilon_0} \frac{qq_0}{r} \]

- This equation is valid no matter what the signs of the charges are.

- Potential energy is defined to be zero when the charges are infinitely far apart.
Graphs of the potential energy

- If two charges have the same sign, the interaction is *repulsive*, and the electric potential energy is *positive*.

\[
U(r) = \frac{kq_0 q}{r} \quad \text{for} \quad r > 0
\]

- As \( r \to 0 \), \( U \to +\infty \).
- As \( r \to \infty \), \( U \to 0 \).
• If two charges have opposite signs, the interaction is *attractive*, and the electric potential energy is *negative*. 

\[
\begin{align*}
U & < 0 \\
\text{As } r \to 0, \ U \to -\infty. \\
\text{As } r \to \infty, \ U \to 0. 
\end{align*}
\]
Potential energy of \( q_0 \) with several point charges

- The potential energy associated with \( q_0 \) depends on the other charges and their distances from \( q_0 \).

- The electric potential energy is the \textit{algebraic sum (linearity)}:

\[
U = \frac{q_0}{4\pi\epsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots \right) = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}
\]

Electric constant
Distances from \( q_0 \) to \( q_1, q_2, q_3, \ldots \)

However this is \textbf{not} the full potential energy of the \textbf{system} of charges. We need to count the contribution of all pairs, avoiding \textbf{double counting}.

\[
U_{sys} = \frac{1}{4\pi\epsilon_0} \sum_{j<i} \frac{q_i q_j}{r_{ij}} = \frac{1}{4\pi\epsilon_0} \frac{1}{2} \sum_{j \neq i} \frac{q_i q_j}{r_{ij}}
\]

\( r_{ij} \) is the distance between charges \( i \) and \( j \).
**Electric potential**

- **Potential** is *potential energy per unit charge*.

- The potential of \( a \) with respect to \( b \) \((V_{ab} = V_a - V_b)\) equals the work done by the electric force when a *unit* charge moves from \( a \) to \( b \).
Electric potential

• The potential due to a single point charge is:

\[ V = \frac{1}{4\pi \epsilon_0} \frac{q}{r} \]

- **Electric potential due to a point charge**
- **Value of point charge**
- **Electric constant**
- **Distance from point charge to where potential is measured**

• Like electric field, potential is independent of the test charge that we use to define it.

• For a collection of point charges:

\[ V = \frac{1}{4\pi \epsilon_0} \sum_i \frac{q_i}{r_i} \]

- **Electric potential due to a collection of point charges**
- **Value of \( i \)th point charge**
- **Electric constant**
- **Distance from \( i \)th point charge to where potential is measured**
Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*.
Electric potential and electric field

• Moving with the direction of the electric field means moving in the direction of decreasing $V$, and vice versa.

• To move a unit charge slowly against the electric force, we must apply an external force per unit charge equal and opposite to the electric force per unit charge.

• The electric force per unit charge is the electric field.

• The potential difference $V_a - V_b$ equals the work done per unit charge by this external force to move a unit charge from $b$ to $a$:

$$V_a - V_b = - \int_b^a \vec{E} \cdot d\vec{l}$$

• The unit of electric field can be expressed as $1 \text{ N/C} = 1 \text{ V/m}$.
The electron volt

• When a particle with charge $q$ moves from a point where the potential is $V_b$ to a point where it is $V_a$, the change in the potential energy $U$ is

$$U_a - U_b = q(V_a - V_b)$$

• If charge $q$ equals the magnitude $e$ of the electron charge, and the potential difference is 1 V, the change in energy is defined as one electron volt (eV):

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$
Electric potential and field of a charged conductor

- A solid conducting sphere of radius $R$ has a total charge $q$.

- The electric field inside the sphere is zero everywhere.

- **Two different cases are mixed here**: If we have uniform surface charge on an insulating sphere $E=0$ inside because of Gauss law and symmetry.

- If we have conductor of any shape, $E=0$ inside because free charges are at rest.

Graph of electric field

$$E = \frac{1}{4\pi \varepsilon_0} \frac{q}{R^2}$$
Electric potential and field of a charged conductor

- The potential is the *same* at every point inside the sphere and is equal to its value at the surface.

\[
V = \frac{1}{4\pi \epsilon_0} \frac{q}{r}
\]

Graph of potential
Oppositely charged parallel plates

- Choose $V(0)=0$. $E$ is constant and points downwards.
- $V(y) = -\int_{0}^{y} \vec{E} \cdot d\vec{r} = -\int_{0}^{y} -Edy = E \int_{0}^{y} dy = Ey$
- The potential at any height $y$ between the two large oppositely charged parallel plates is $V = Ey$. $y$
Equipotential surfaces

• Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.
Equipotential surfaces and field lines

- An **equipotential surface** is a surface on which the electric potential is the same at every point.

- Field lines and equipotential surfaces are always mutually perpendicular.

- Shown are cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for a single positive charge.
Equipotential surfaces and field lines for a dipole

\[ V = -30 \text{ V} \]
\[ V = -50 \text{ V} \]
\[ V = 0 \text{ V} \]
\[ V = +30 \text{ V} \]
\[ V = -70 \text{ V} \]
\[ V = +70 \text{ V} \]

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