Lecture 12: Magnetic Forces (cont’d)

Ch. 27: Sources of the magnetic field

Looking forward at …

- how to analyze magnetic forces on current-carrying conductors.
- Loop of current is a magnetic dipole
- DC electric motors
- what is the fundamental reason for magnetic fields
- how to calculate the magnetic field produced by a single moving charged particle, a straight current-carrying wire, or a current-carrying wire bent into a circle.
- why wires carrying current in the same direction attract, while wires carrying opposing currents repel.

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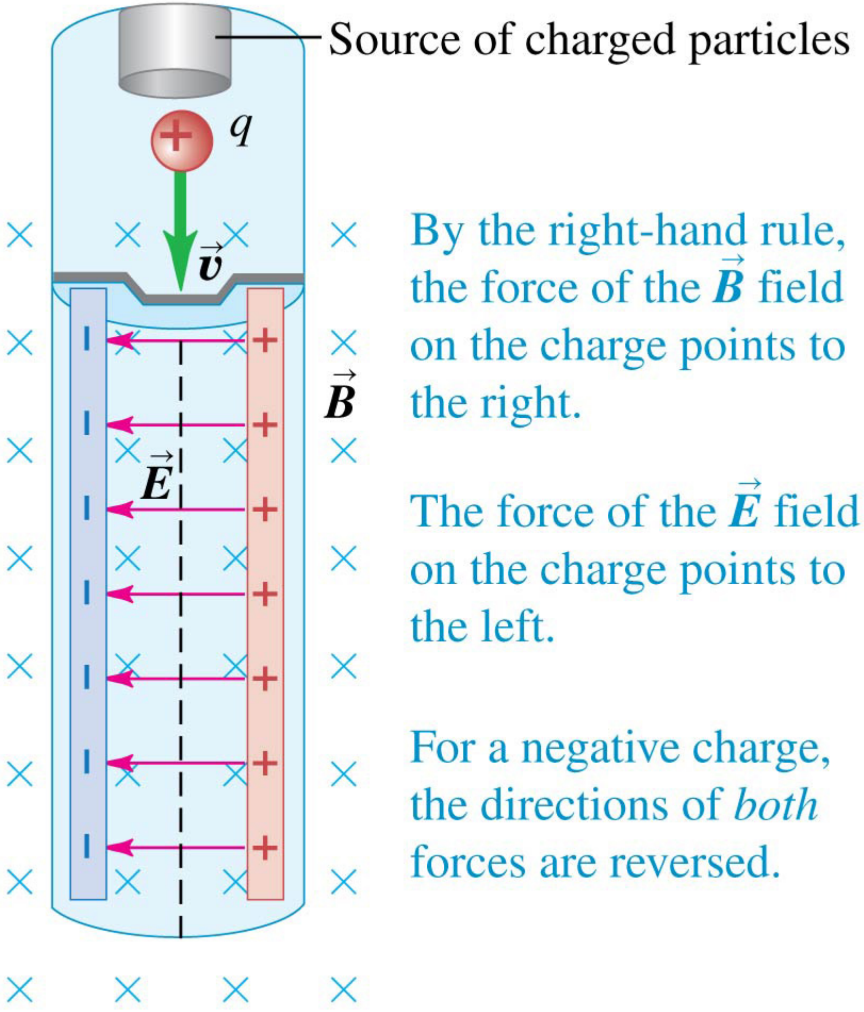
Velocity selector

- A **velocity selector** uses perpendicular electric and magnetic fields to select particles of a specific speed from a beam.
- Only particles having speed $v = E/B$ pass through undeflected.



By the right-hand rule, the force of the $\vec{B}$ field on the charge points to the right.

The force of the $\vec{E}$ field on the charge points to the left.

For a negative charge, the directions of *both* forces are reversed.

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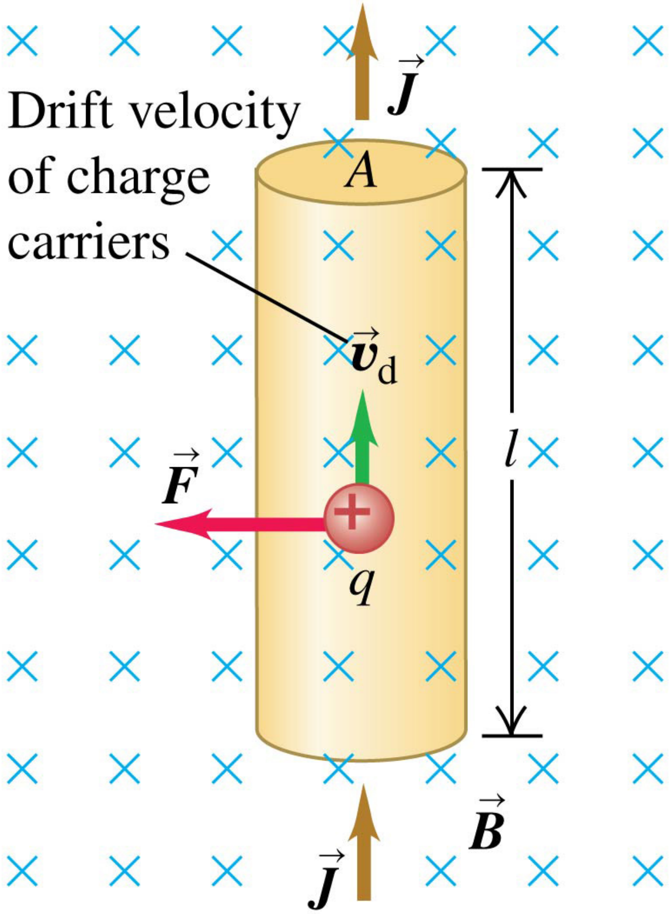
The magnetic force on a current-carrying conductor

- The figure shows a straight segment of a conducting wire, with length *l* and cross-sectional area *A*.
- The magnitude of the force on a single charge is $F = qv_dB$.
- If the number of charges per unit volume is *n*, then the total force on all the charges in this segment is

$$F = (nAl)(qv_dB) = (nqv_d A)(lB) = IlB$$

A more formal way to get the force on a ***small length dl*** of the conductor:

$$\overrightarrow{dF} = dq\vec{v}\times\vec{B} = dq\frac{\overrightarrow{dl}}{dt}\times\vec{B} = \frac{dq}{dt}\overrightarrow{dl}\times\vec{B} = I\,\overrightarrow{dl}\times\vec{B}$$



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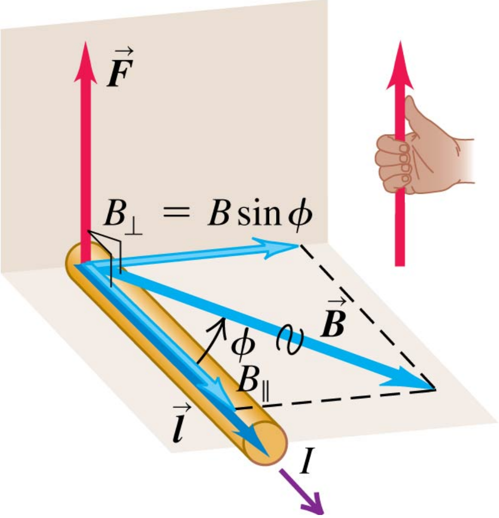
The magnetic force on a current-carrying conductor

- The force is always perpendicular to both the conductor and the field, with the direction determined by the same right-hand rule we used for a moving positive charge.

Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle ϕ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_\perp = IlB \sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.



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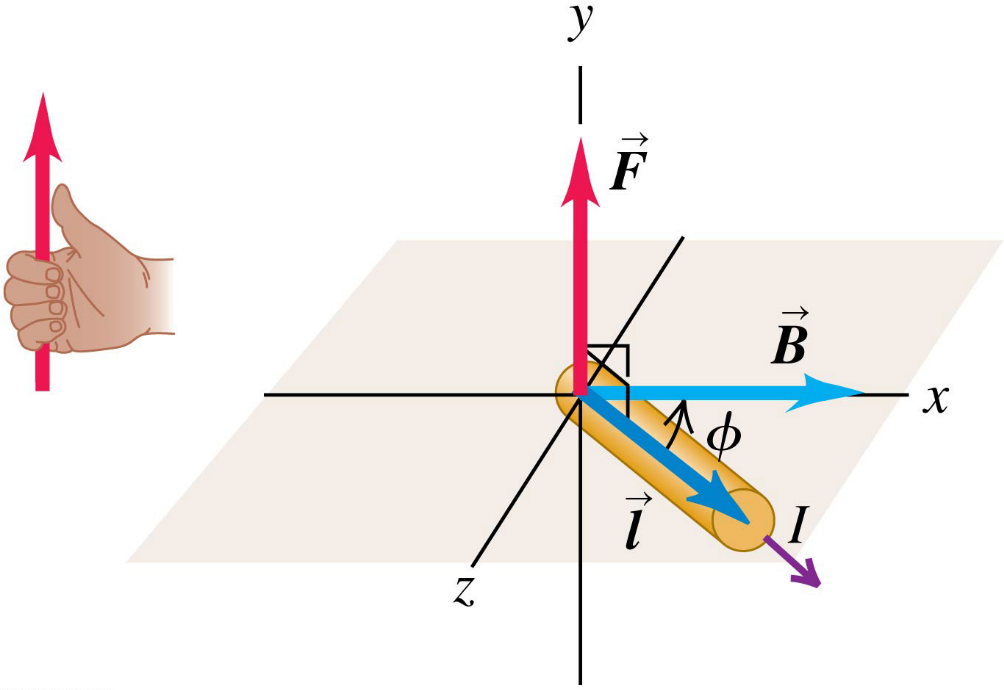
The magnetic force on a current-carrying conductor

- The magnetic force on a segment of a straight wire can be represented as a vector product.



Current

Magnetic force on a straight wire segment $\vec{F} = I\vec{l} \times \vec{B}$ Magnetic field

Vector length of segment (points in current direction)

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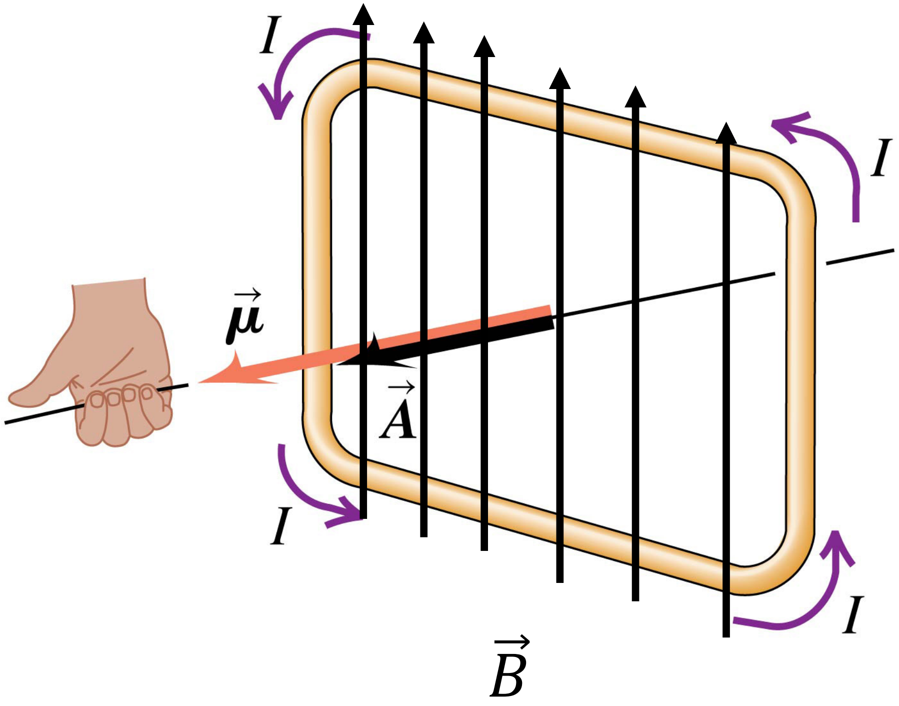
Force and torque on a current loop

- The net force on a current loop in a *uniform* magnetic field is zero.
- $\tau = 2\frac{b}{2}F = bIaB = IAB$
- We can define a magnetic moment $\vec{\boldsymbol{\mu}}$ with magnitude *IA*, and direction as shown.
- The net torque on the loop is given by the vector product (maximum torque when $\vec{\mu} \perp \vec{B}$)



Vector magnetic torque on a current loop $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\boldsymbol{B}}$ Magnetic dipole moment; Magnetic field

- $U_m = -\vec{\mu} \cdot \vec{B}$

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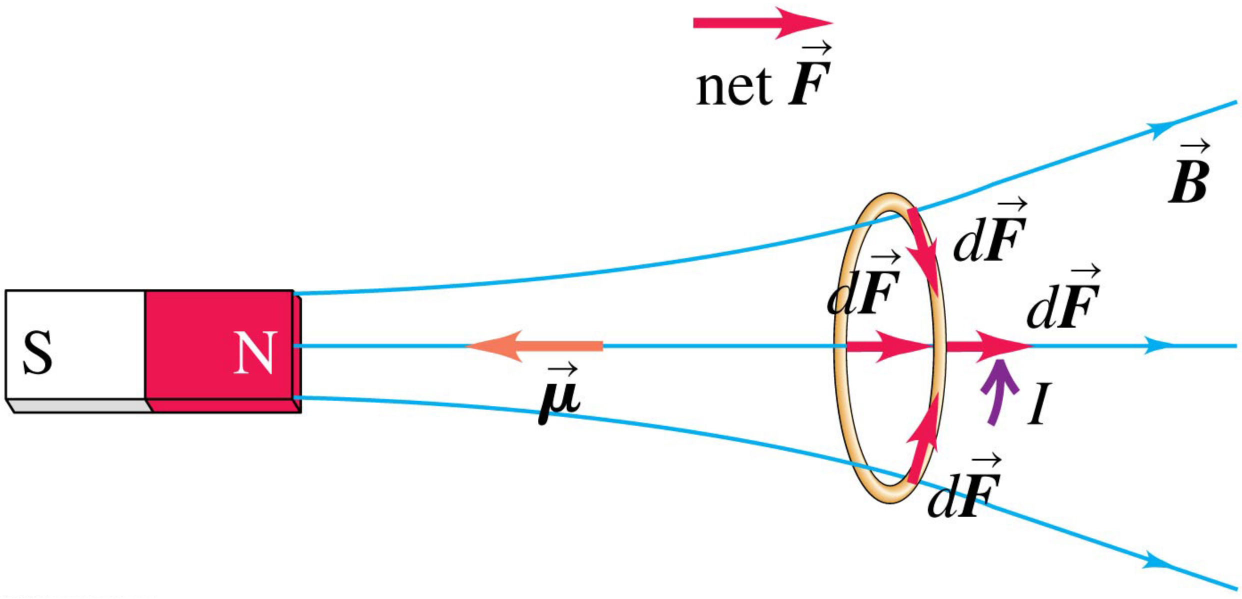
Magnetic dipole in a nonuniform magnetic field

- A current loop with magnetic moment pointing to the left is in a magnetic field that decreases in magnitude to the right.
- When these forces are summed to find the net force on the loop, the radial components cancel so that the net force is to the right, away from the magnet.



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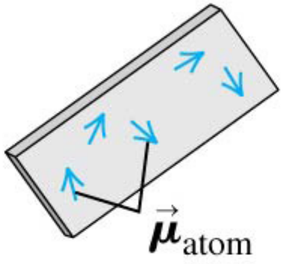
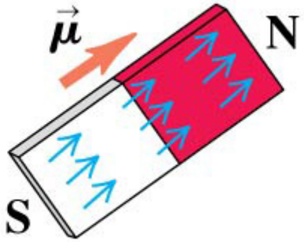
How magnets work: pre-existing dipoles

- **(a) An unmagnetized piece of iron**. Only a few representative atomic moments are shown.
- **(b) A magnetized piece of iron** (bar magnet). The net magnetic moment of the bar magnet points from its south pole to its north pole.

(a) Unmagnetized iron: magnetic moments are oriented randomly.



(b) In a bar magnet, the magnetic moments are aligned.



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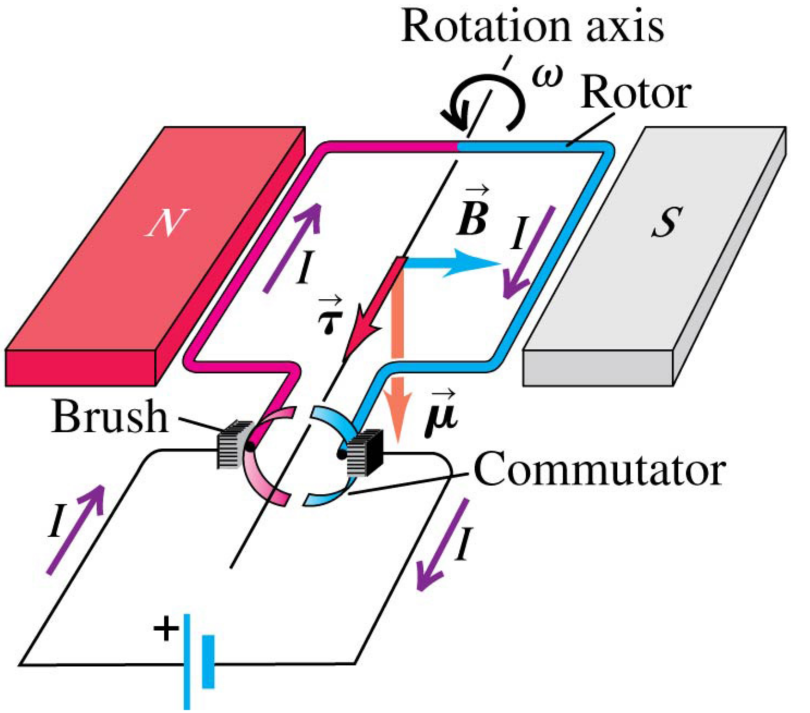
The direct-current motor

- Below is a schematic diagram of a simple dc motor.
- The **rotor** is a wire loop that is free to rotate about an axis; the rotor ends are attached to the two curved conductors that form the **commutator**.
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.



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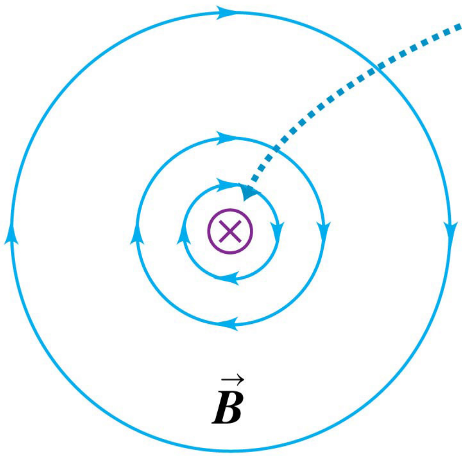
Ch. 27 :The magnetic field of a moving charge

- **A moving charged particle generates a magnetic field** that depends on its velocity, its charge and the distance from it.

View from behind the charge



The × symbol indicates that the charge is moving into the plane of the page (away from you).

- Very similar to Coulomb's law:

Magnetic field due to a point charge with constant velocity

Magnetic constant

Charge

Velocity

Unit vector from point charge toward where field is measured

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

Distance from point charge to where field is measured

Don't confuse this formula with the magnetic force $\vec{F} = q\vec{v}\times\vec{B}$, which tells how a moving charge reacts to a magnetic field!

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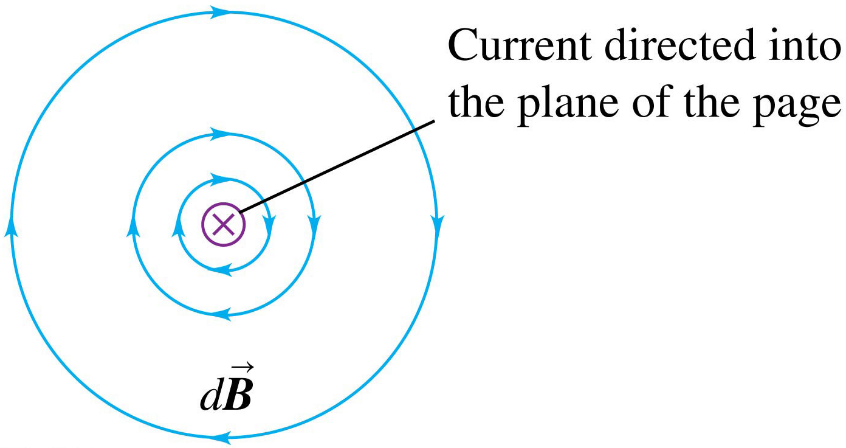
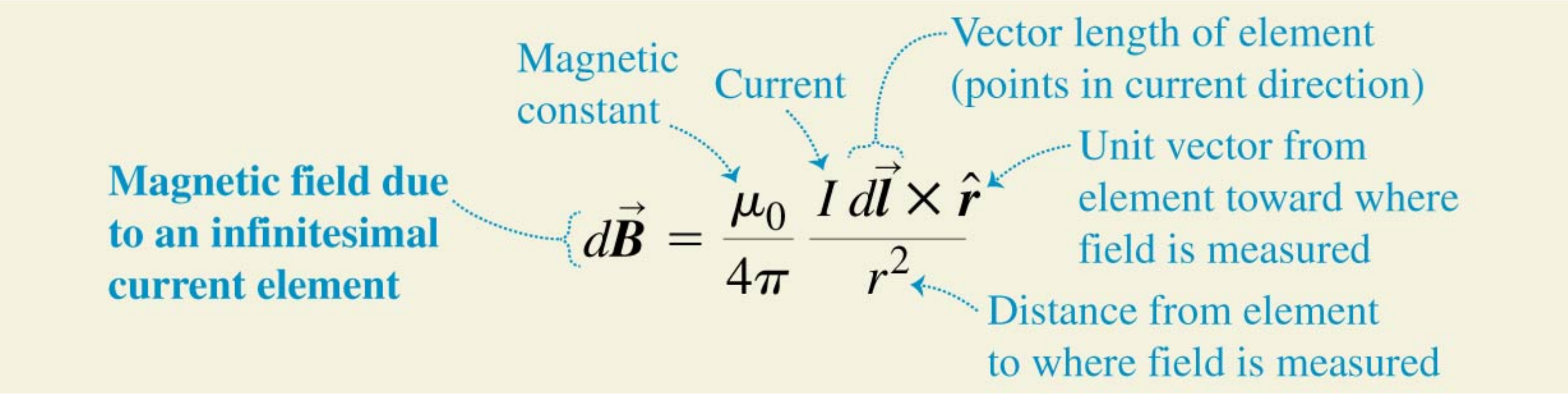
Magnetic field of a current element

- The total magnetic field of several moving charges is the vector sum of each field.
- The magnetic field caused by a short segment of a current-carrying conductor is found using the **law of Biot and Savart**:

View along the axis of the current element





$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

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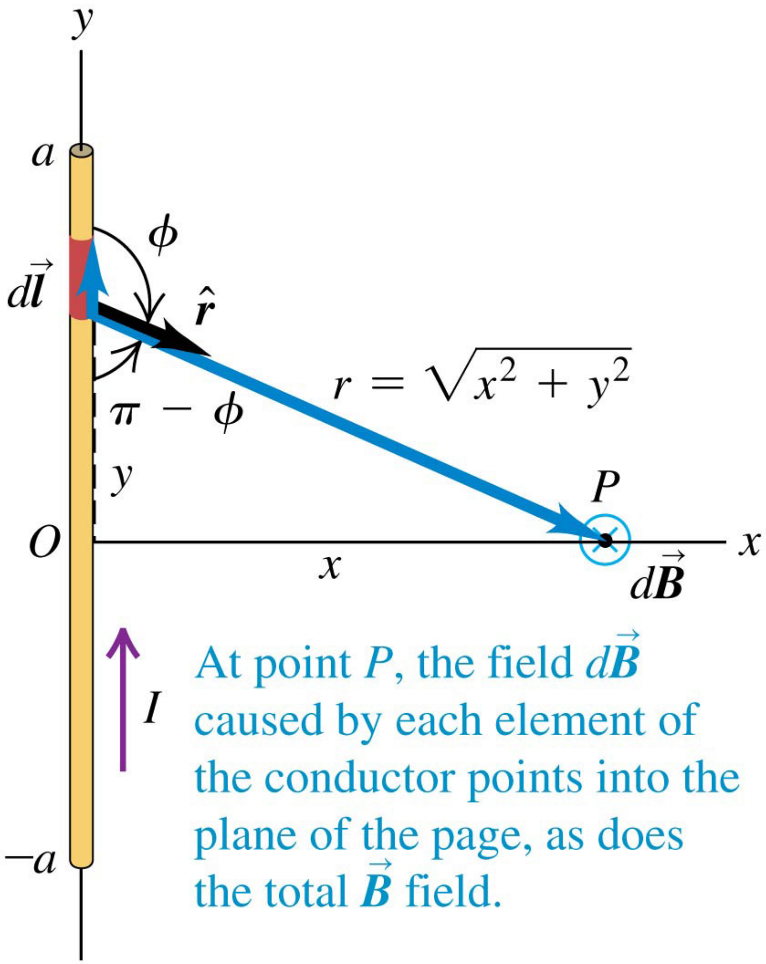
Magnetic field of a straight current-carrying conductor

- Let's use the law of Biot and Savart to find the magnetic field produced by a straight current-carrying conductor.
- The figure shows such a conductor with length 2*a* carrying a current *I*.
- We will find $\vec{B}$ at a point a distance *x* from the conductor on its perpendicular bisector.
- $\vec{B} = I\hat{k}\int_{-a}^{a}\frac{dy}{r^2}\sin(\phi), \quad \sin(\phi) = \frac{x}{r},$
- $\vec{B} = I\hat{k}\int_{-a}^{a}\frac{x\,dy}{(x^2+y^2)^{3/2}},$ we had a similar integral for E previously!



At point *P*, the field $d\vec{B}$ caused by each element of the conductor points into the plane of the page, as does the total $\vec{B}$ field.

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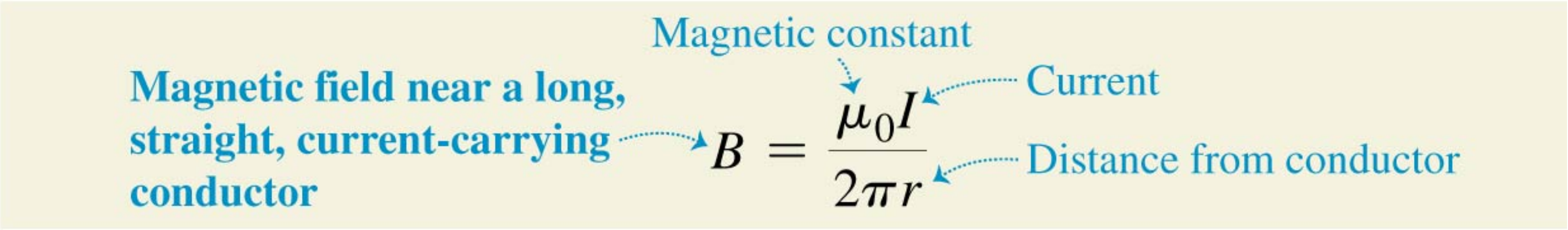
Magnetic field of a straight current-carrying conductor

- Since the direction of the magnetic field from all parts of the wire is the same, we can integrate the magnitude of the magnetic field and obtain:

$$B = \frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}}$$

- As the length of the wire approaches infinity, $x >> a$, and the distance x may be replaced with r to indicate this is a radius of a circle centered on the conductor:



Magnetic constant

Magnetic field near a long, straight, current-carrying conductor

$$B = \frac{\mu_0 I}{2\pi r}$$

Current

Distance from conductor

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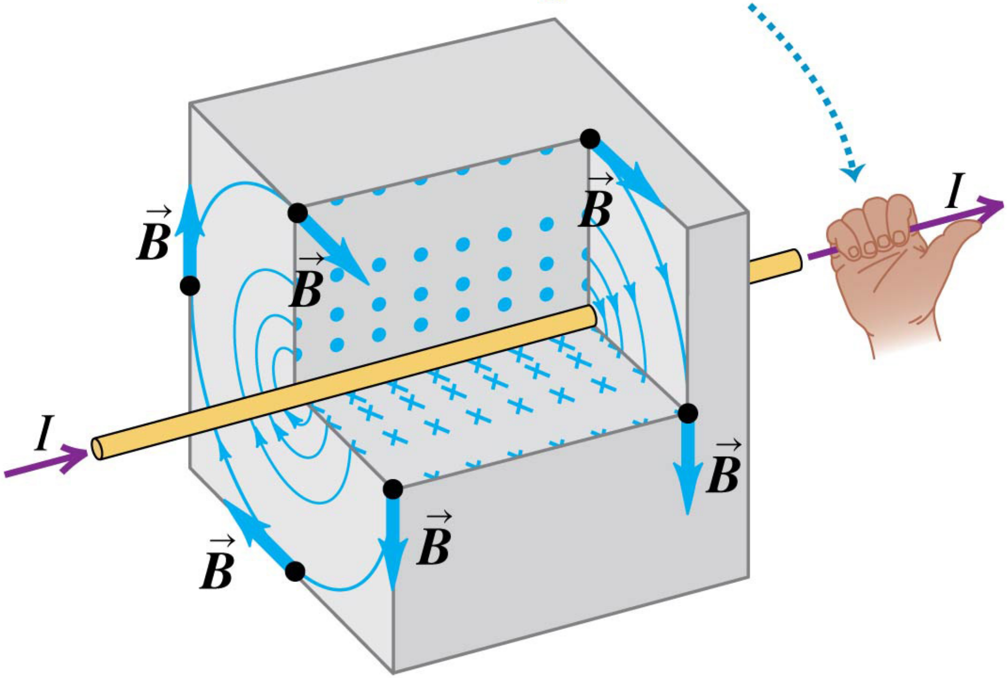
Magnetic field of a straight current-carrying conductor

- The field lines around a long, straight, current-carrying conductor are circles, with directions determined by the right-hand rule.

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



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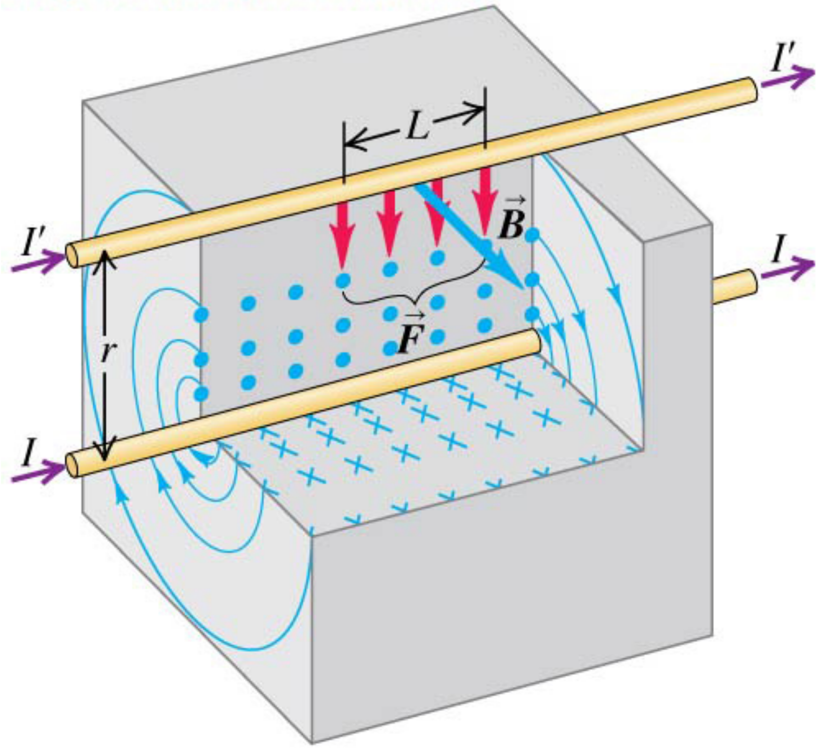
Force between parallel conductors

- The magnetic field of the lower wire exerts an *attractive* force on the upper wire.
- If the wires had currents in *opposite* directions, they would *repel* each other.



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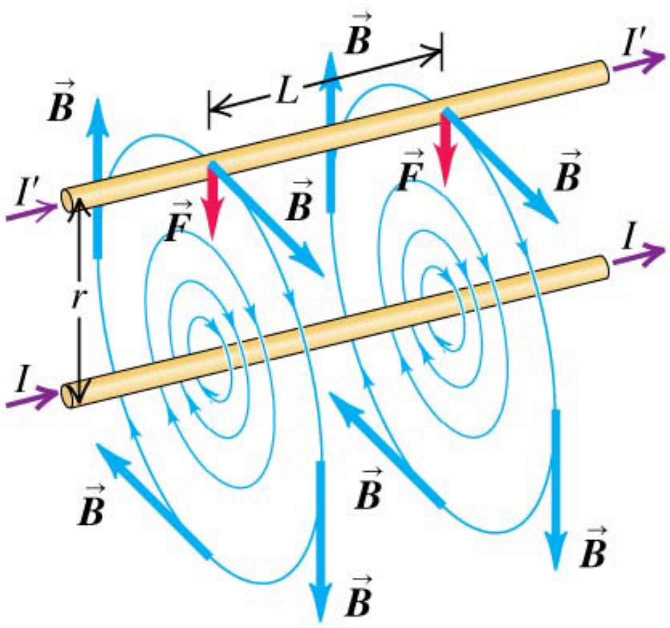
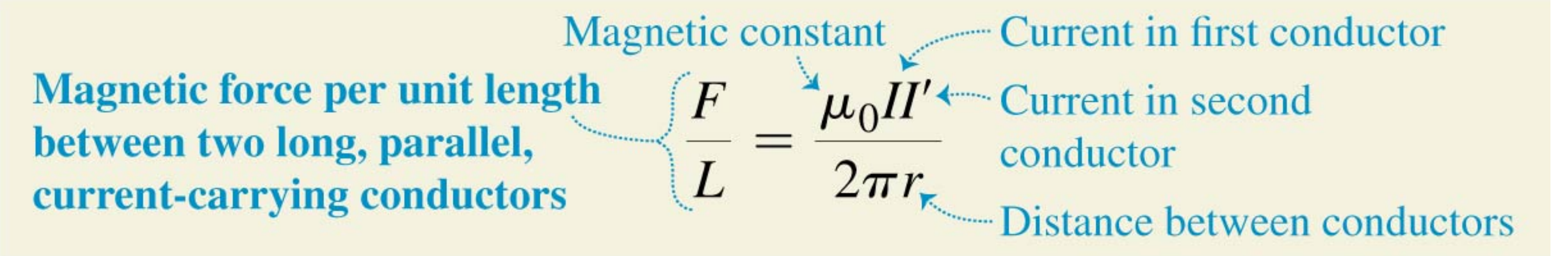
Force between parallel conductors

- The figure shows segments of two long, straight, parallel conductors separated by a distance r and carrying currents I and I' in the same direction.

- Each conductor lies in the magnetic field set up by the other, so each experiences a force.





Magnetic constant

Current in first conductor

Magnetic force per unit length between two long, parallel, current-carrying conductors

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

Current in second conductor

Distance between conductors

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The old definition of the ampere

- The SI definition of the **ampere** is:

> **One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} Newtons per meter of length.**

- This definition of the ampere is what leads us to choose the value of $4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ for the magnetic constant, μ_0.
- The SI definition of the **Coulomb** is the amount of charge transferred in one second by a current of one ampere.
- Since last year the Coulomb is the primary unit and Amperre is C/s.

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Magnetic and electric fields are linked

- Moving charged particles create both electric and magnetic fields.
- Charged particles moving in electric and magnetic fields experience electric and magnetic forces.
- If two charged particles travel with the same constant velocity, in their reference frame they will experience only electric forces.
- To have the Newton's second law the same in the new reference frame, both the electric and magnetic fields should transform in such a way, that each of them has electric and magnetic components in the other reference frame.
- Magnetic fields and electric fields are different manifestations of a unified Electromagnetic field! In some reference frames, there can be only electric fields, in others only magnetic fields.

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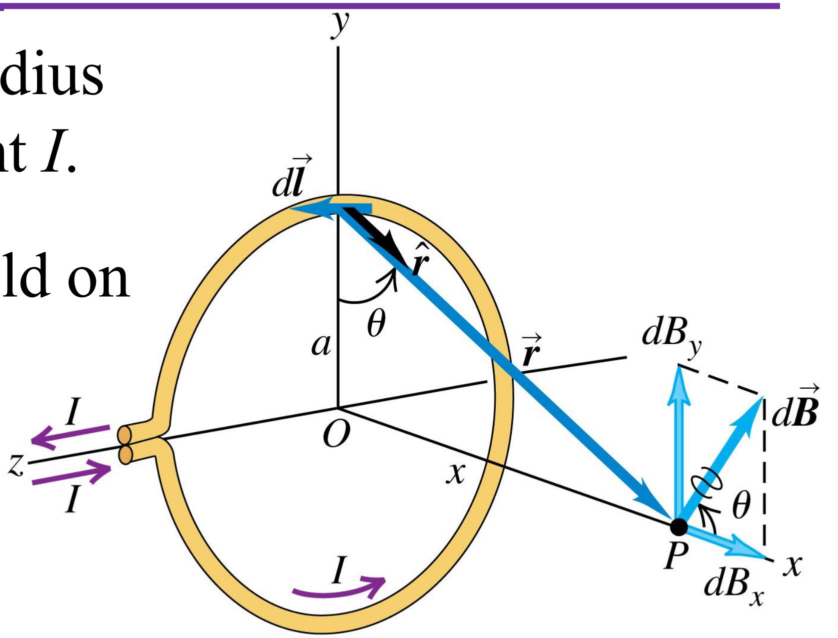
Magnetic field of a circular current loop



- Shown is a circular conductor with radius *a* carrying a counter-clockwise current *I*.
- We wish to calculate the magnetic field on the axis of the loop.
- $\vec{B}_\perp = 0$, diametrically opposite elements cancel their contributions.
- $B_x = \oint \frac{\mu_0}{4\pi} \frac{|I d\vec{l} \times \hat{r}|}{r^2} cos\theta = \frac{\mu_0}{4\pi} \oint I \; \frac{dl}{r^2} \frac{a}{r}$
- All quantities in the integral are constants! $\oint dl = 2\pi a$
- $B_x = \frac{\mu_0}{4\pi} I \frac{2\pi a^2}{r^3}, \quad r = \sqrt{x^2 + a^2}$

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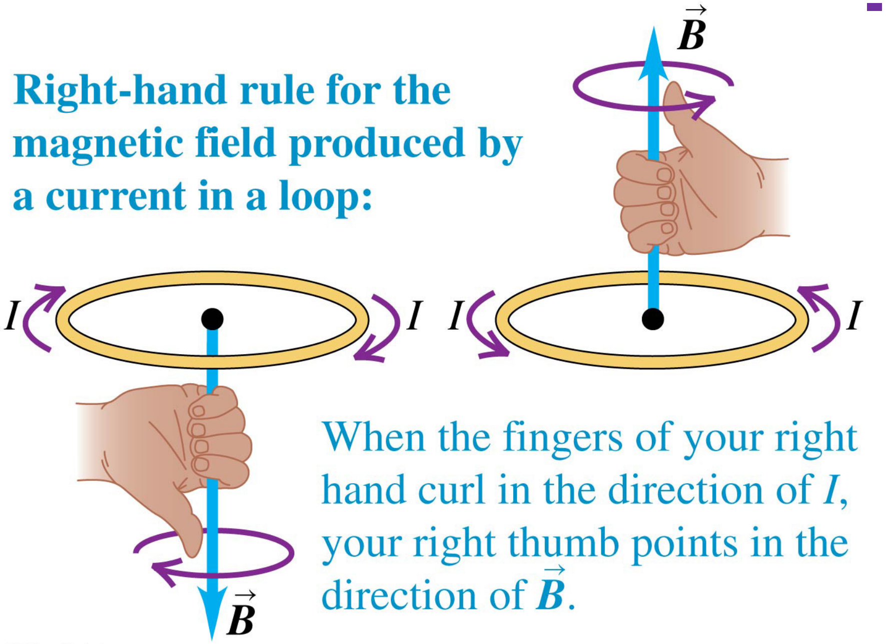
Magnetic field of a circular current loop

- The magnetic field along the axis of a loop of radius *a* carrying a current *I* is given by the equation below.

- The direction is given by the right-hand rule shown.



Right-hand rule for the magnetic field produced by a current in a loop:

When the fingers of your right hand curl in the direction of I, your right thumb points in the direction of $\vec{B}$.

Magnetic field on axis of a circular current-carrying loop

Magnetic constant

Current

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Radius of loop

Distance along axis from center of loop to field point

- $\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi(x^2+a^2)^{3/2}}$, for $x \gg a$ $\qquad \vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{x^3}$

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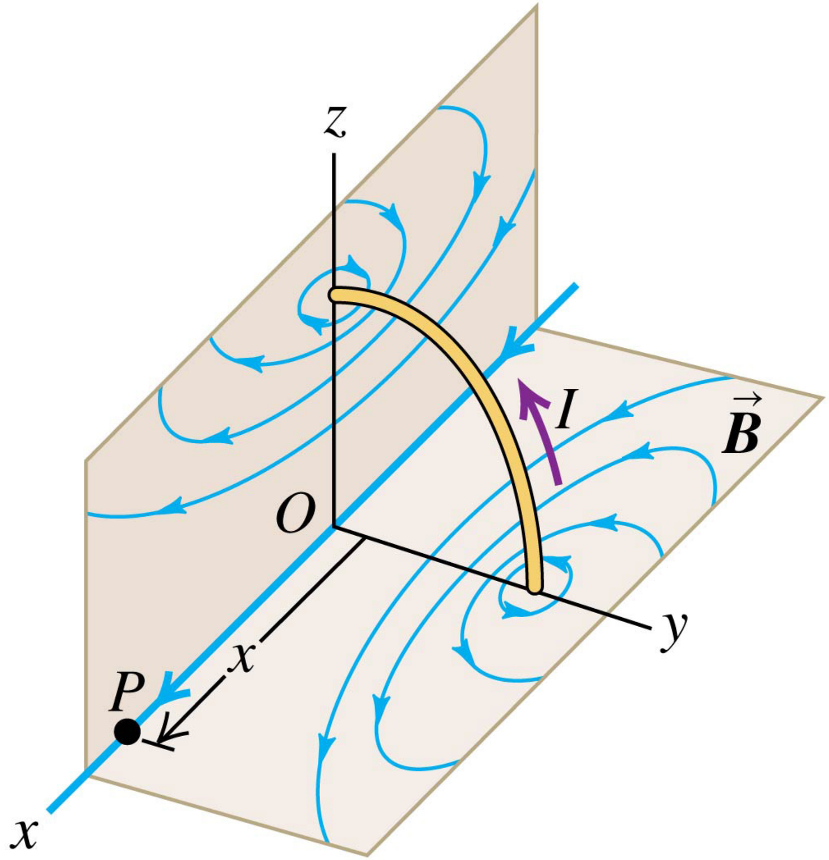
Magnetic field lines of a circular current loop

- The figure shows some of the magnetic field lines surrounding a circular current loop (magnetic dipole) in planes through the axis.
- The field lines for the circular current loop are closed curves that encircle the conductor; they are not circles, however.



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Magnetic fields for MRI

- MRI (magnetic resonance imaging) requires a magnetic field of about 1.5 T.
- In a typical MRI scan, the patient lies inside a coil that produces the intense field.
- The currents required are very high, so the coils are bathed in liquid helium at a temperature of 4.2 K to keep them from overheating.

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